

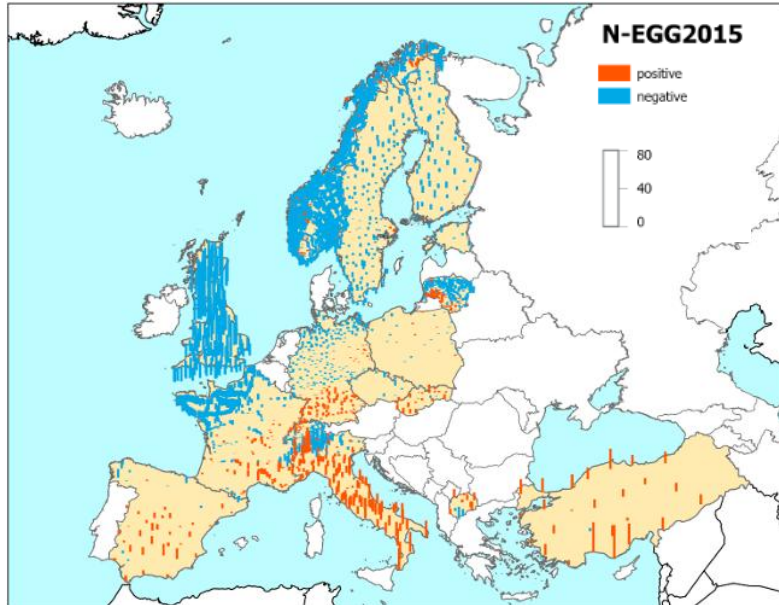
Modern Concepts and Quantum Technology for Geodesy

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Institute of Geodesy, Leibniz University Hannover
Germany

Why Better Measurements Using Quantum Technology and Relativity?

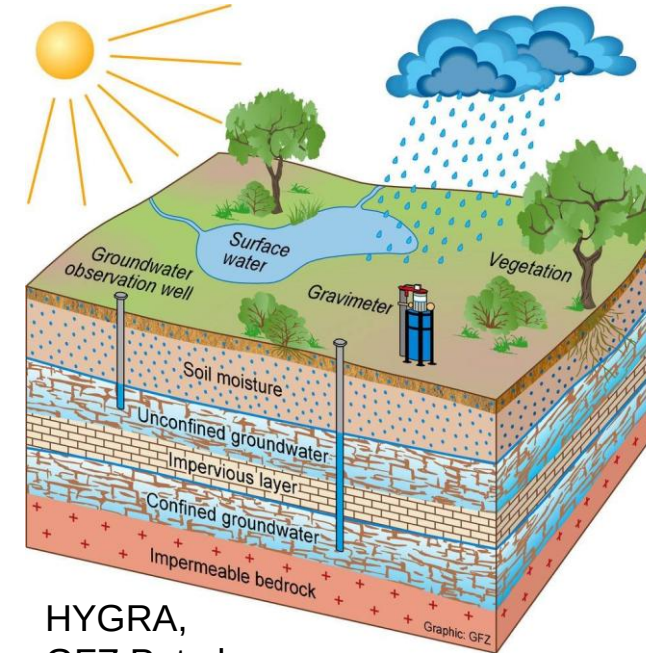
Errors in height systems



$h(eII) - H^N$ (EVRF2019) – EGG2015

[Sacher and Schwabe (2023), BKG]

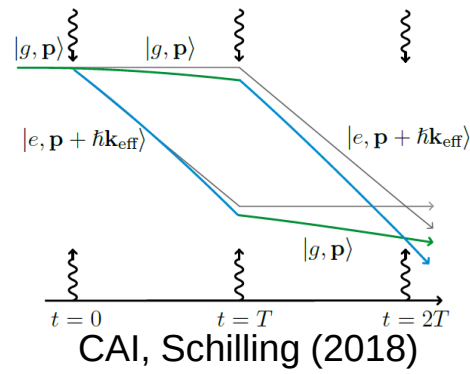
Shortcomings in knowing the time variable gravity field



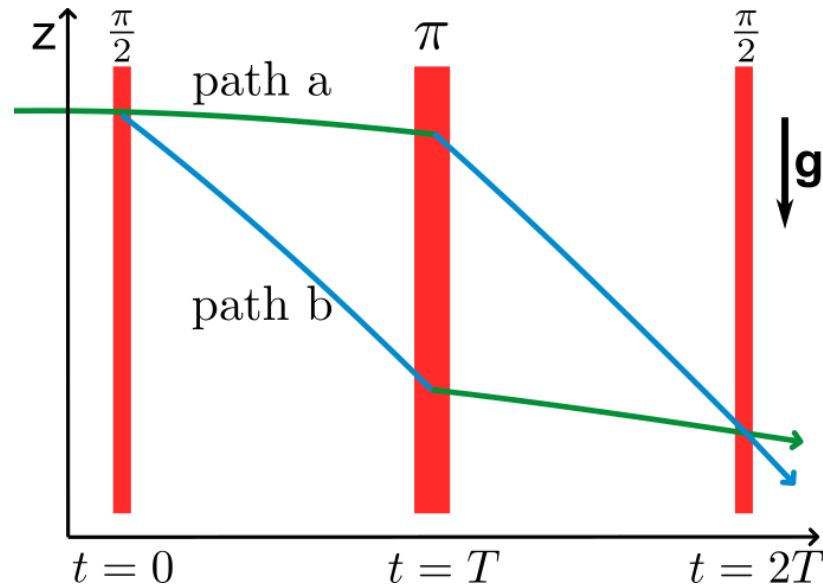
HYGRA,
GFZ Potsdam

Geodesy has to provide reliable **physical reference** and support understanding **processes in the Earth system**

Quantum Gravimetry



Gravimetry Using Cold Atom Interferometry (CAI)



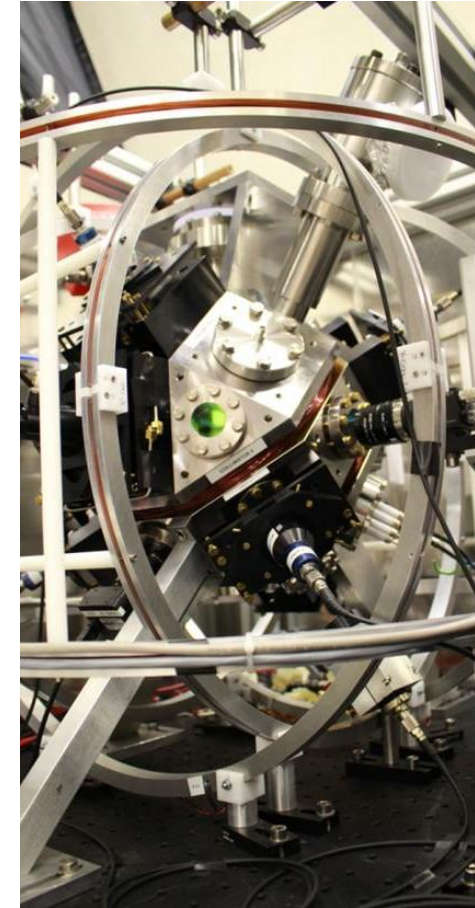
Scheme of a light-pulse atom interferometer

$$g = \frac{\Delta\varphi}{k_{eff}T^2}$$

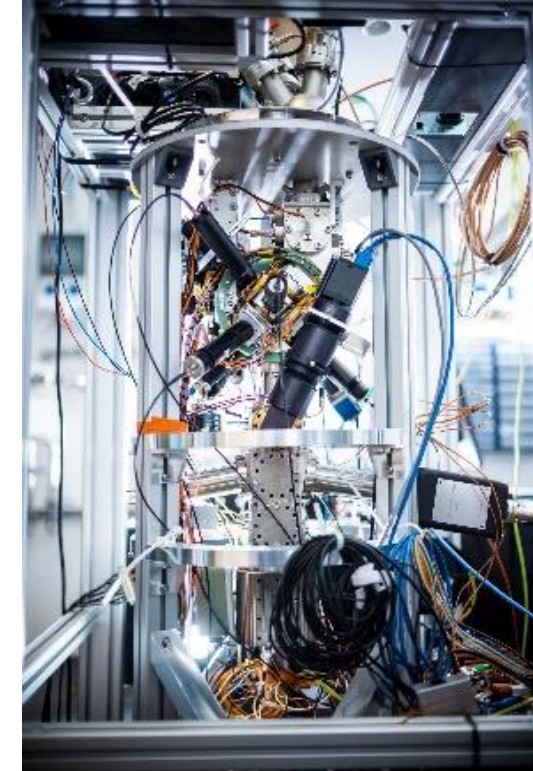
signal: 9.8...m/s²

$\sigma \sim 10^{-8}$ m/s²

$\sigma \sim 10^{-9} - 10^{-12}$ m/s² in space



Quantum accelerometer
Lévêque et al. (2023)

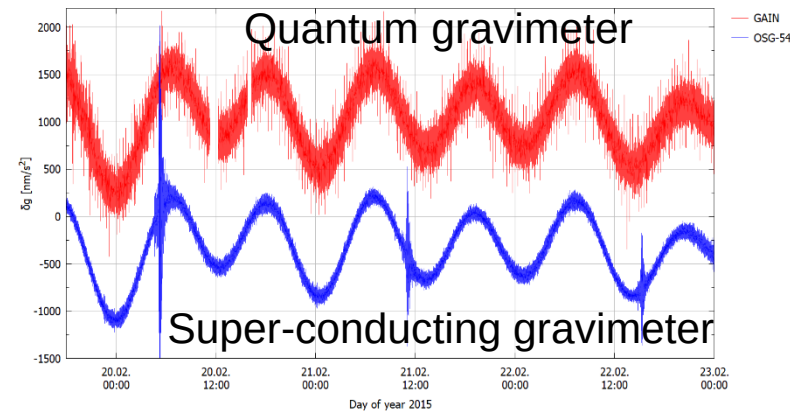
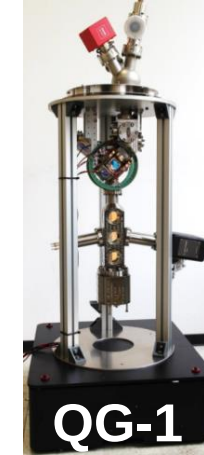
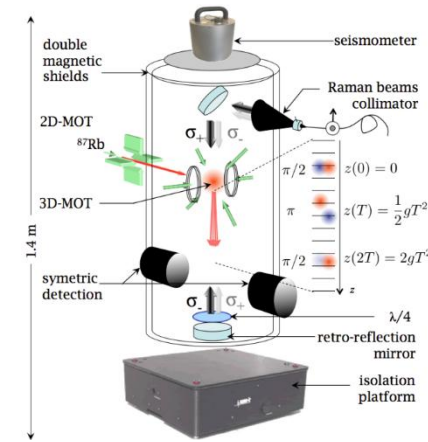
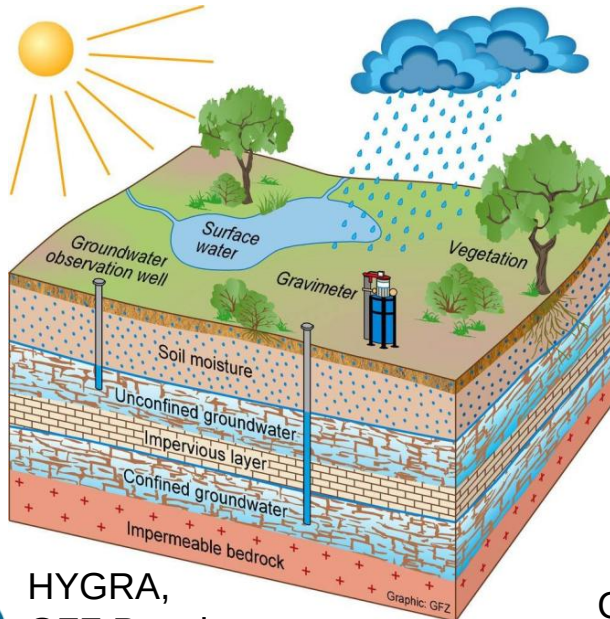


Quantum gravimeter QG-1,
IQO, LUH

Quantum Gravimetry on Ground – Added-value

Atomic gravity sensors can achieve an accuracy of better than 3 nm/s^2 within 1 h

- gravity mirror into underground
- fast surveys, dense coverage
- also, use as registration device
- and stationary, the VLBAI as new gravity reference



Recent EU Project EQUIP-G to Show the Benefit of Quantum Sensors in selected use cases

Investigate many pressing European societal challenges

- 1) Climate change monitoring
- 2) Nuclear waste underground storage monitoring
- 3) Continuous or time-lapse volcano monitoring
- 4) Geothermal energy
- 5) Water storage assessment
- 6) Geodesy and national spatial references

EXAIL quantum gravimeters

- Transportable in car
- Outdoor deployment possible
- Precision good (further improvement expected)
- Setup teachable for non-quantum experts

(1)



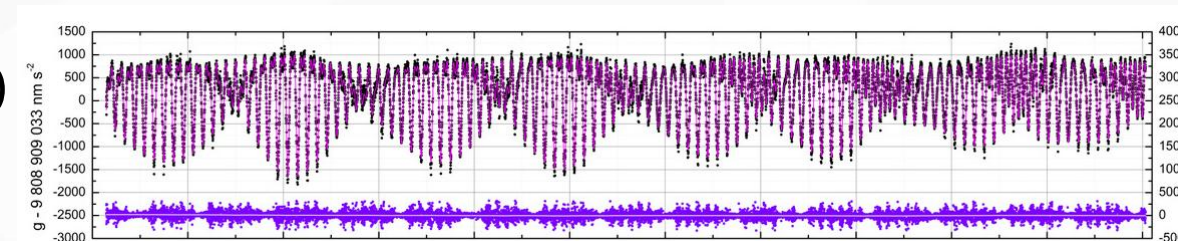
(3)



(2)



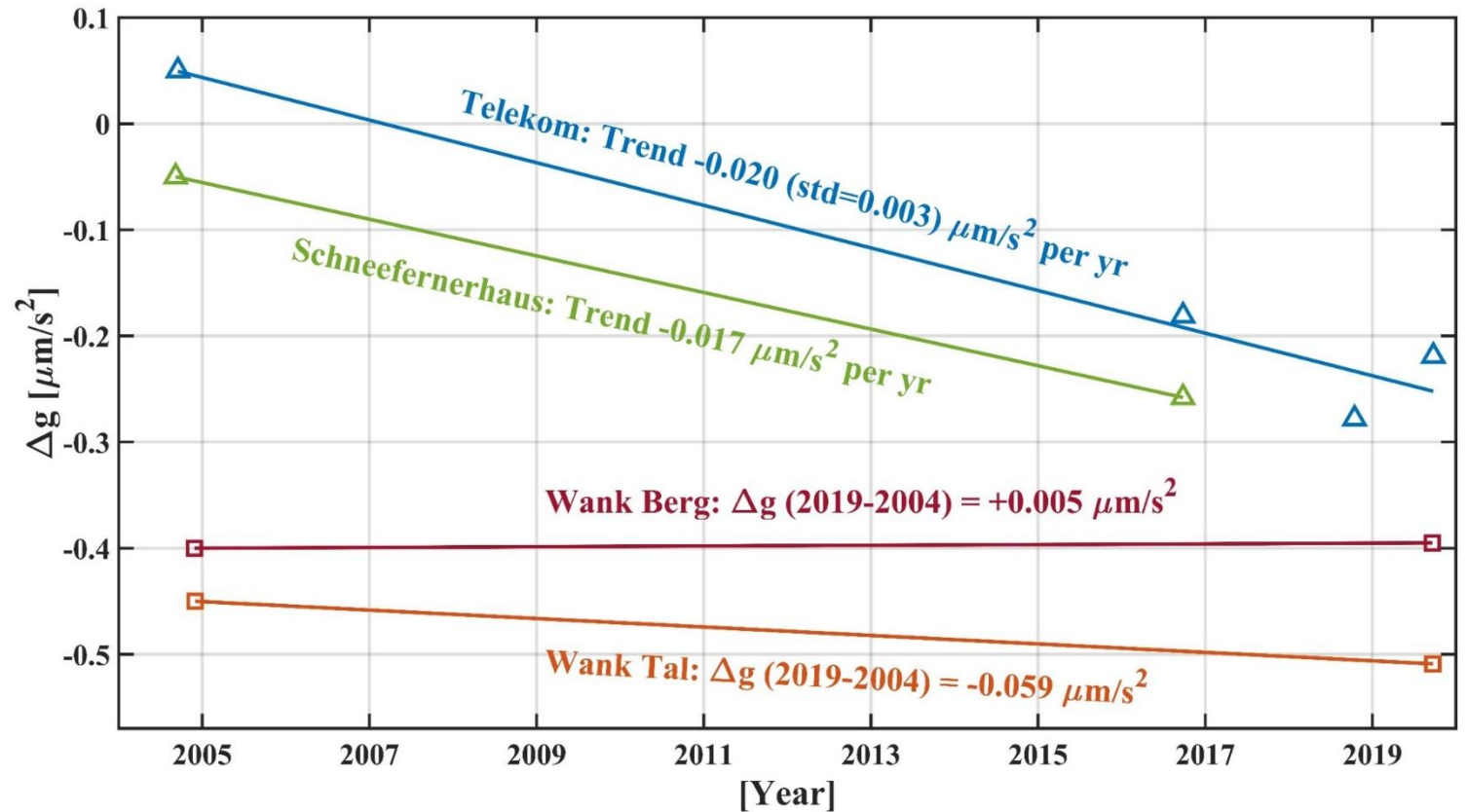
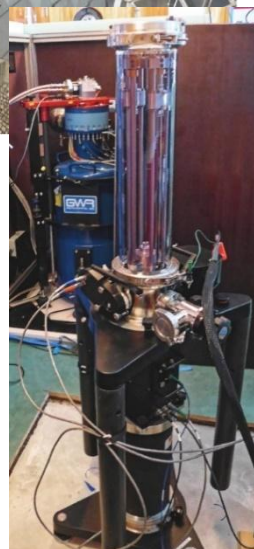
(4)



Long-term Gravity Observations at Zugspitze



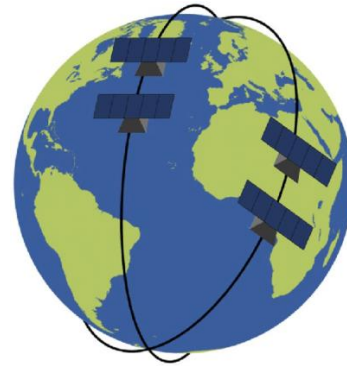
Absolute gravity observations at Mt. Zugspitze since 2004



Obtained gravity variations at Mt. Zugspitze (min. snow coverage):
 $\approx 0.30 \mu\text{m/s}^2$ (2019–2004), std. uncertainty: $0.04 \mu\text{m/s}^2$

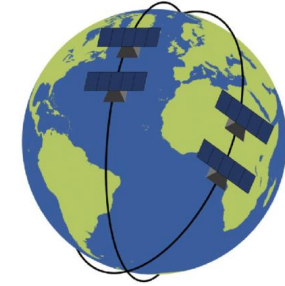
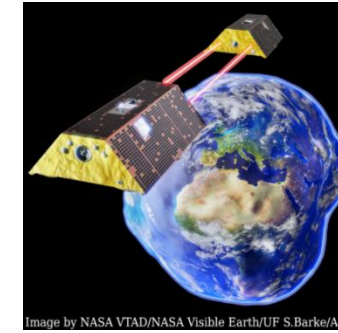
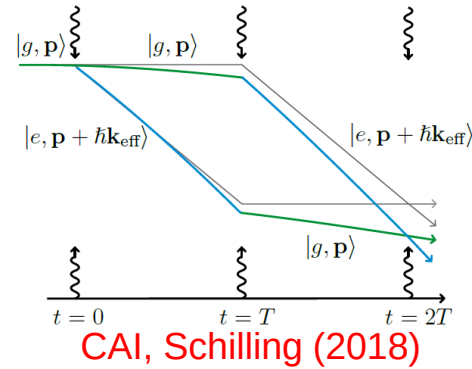
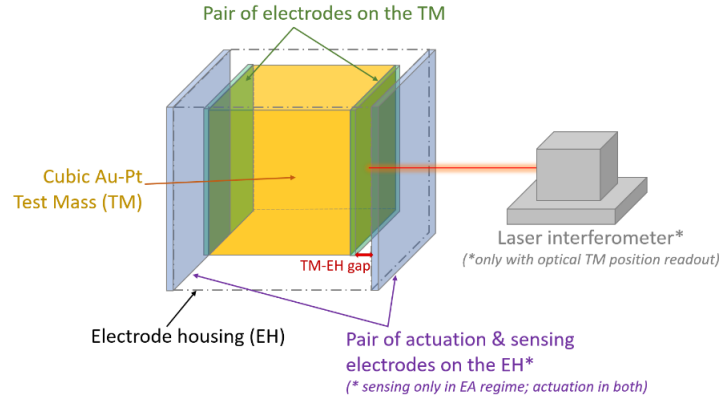
- 60% of the observed g -change at Zugspitze ($-0.02 \mu\text{m/s}^2$ per year) is explained by glacier melting
- What is going-on inside the mountain?

Quantum Gravimetry from Space

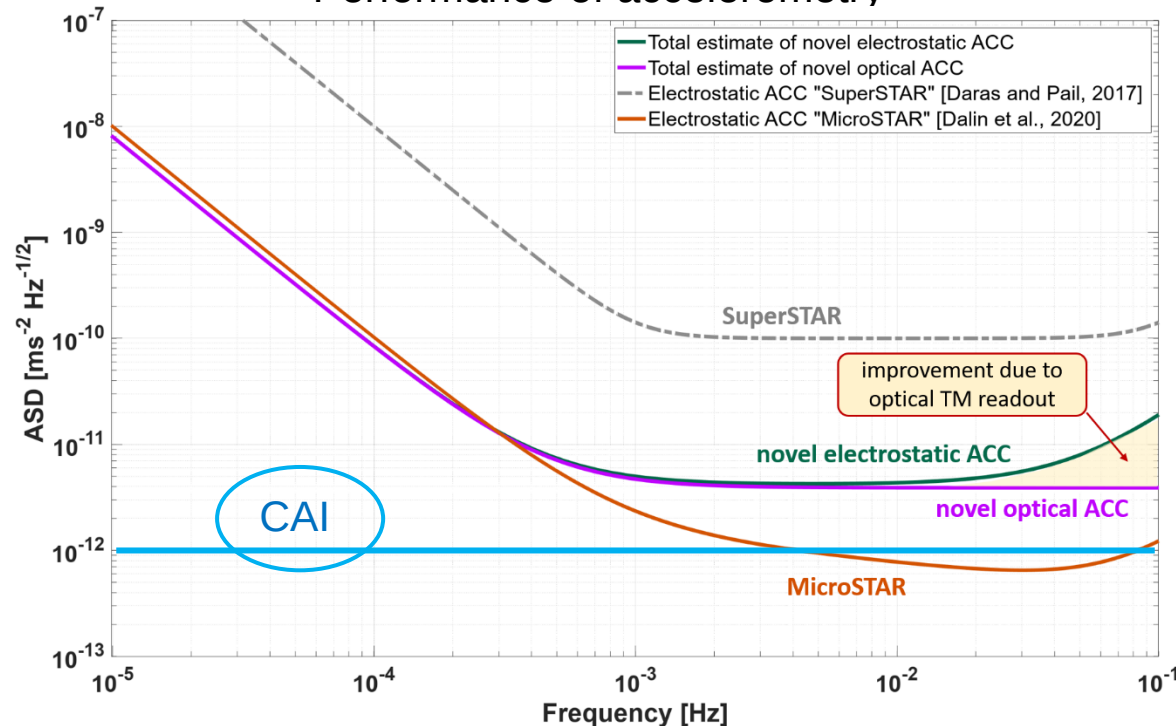


Pail, TUM

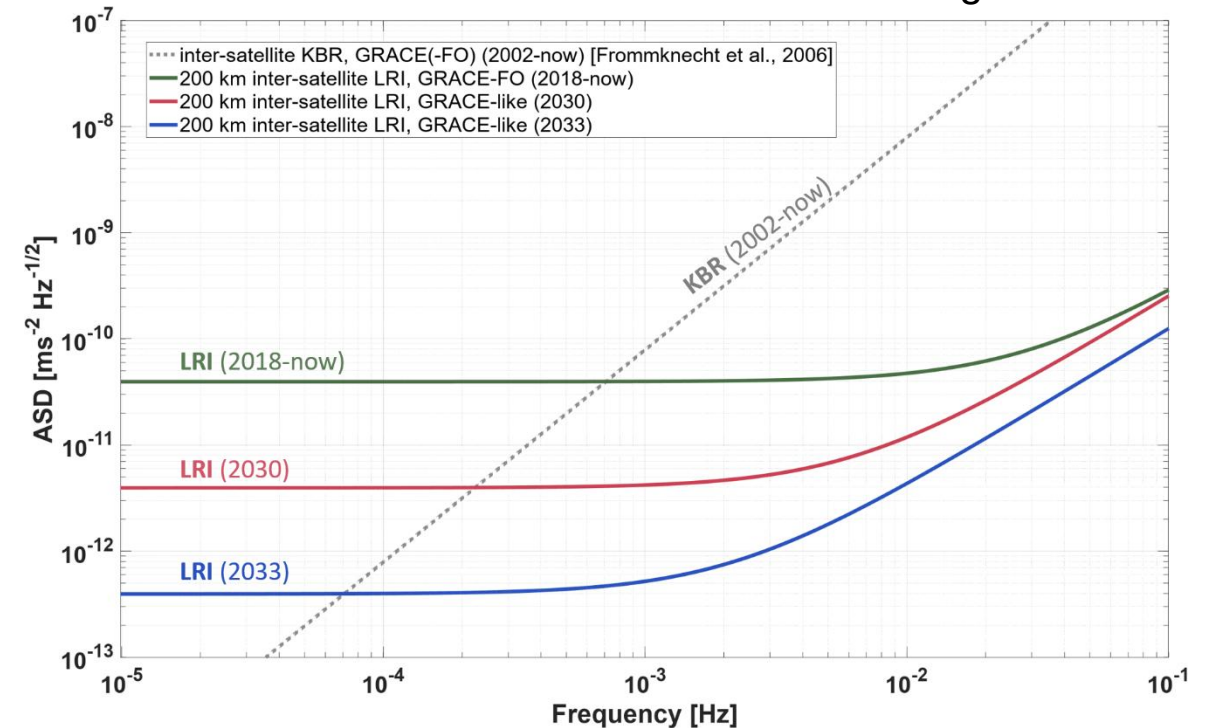
Performance of Accelerometers and Inter-satellite Tracking



Performance of accelerometry



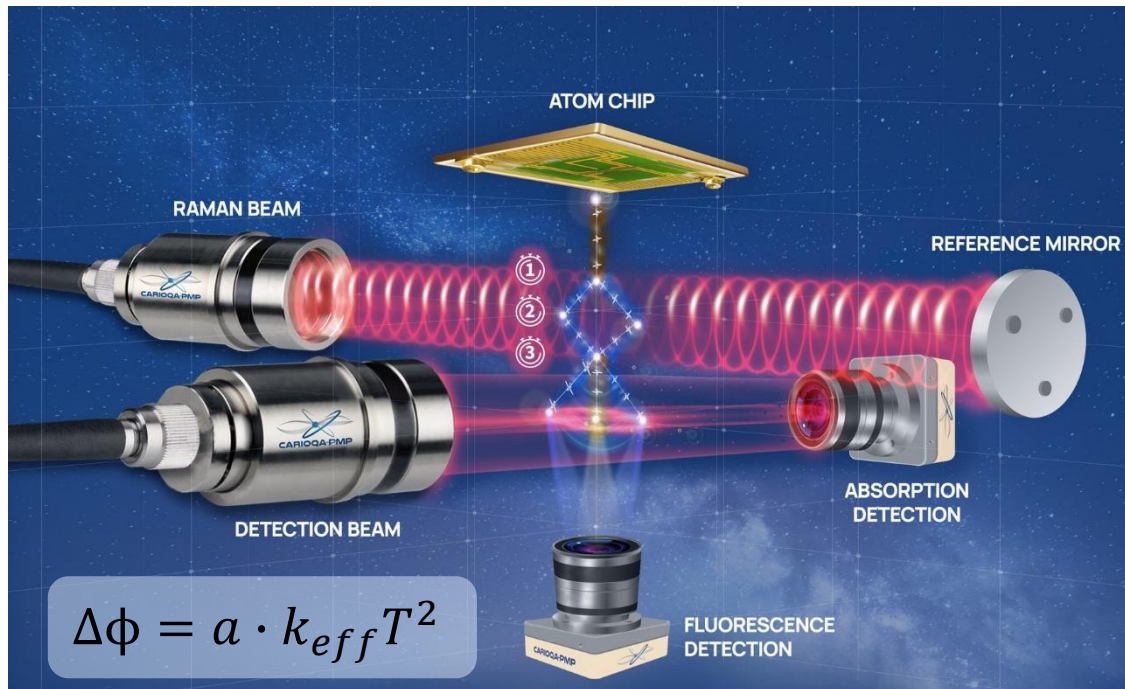
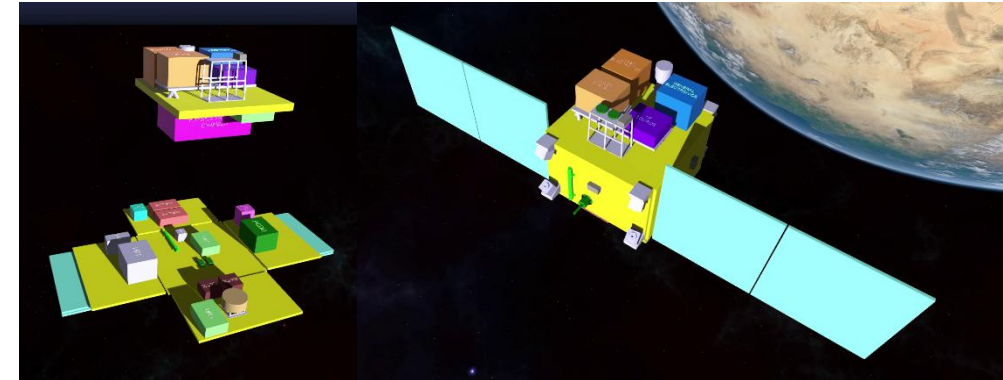
Performance of inter-satellite tracking



CARIOQA Pathfinder Mission Definition

Cold Atom Rubidium Interferometer in Orbit for Quantum Accelerometry

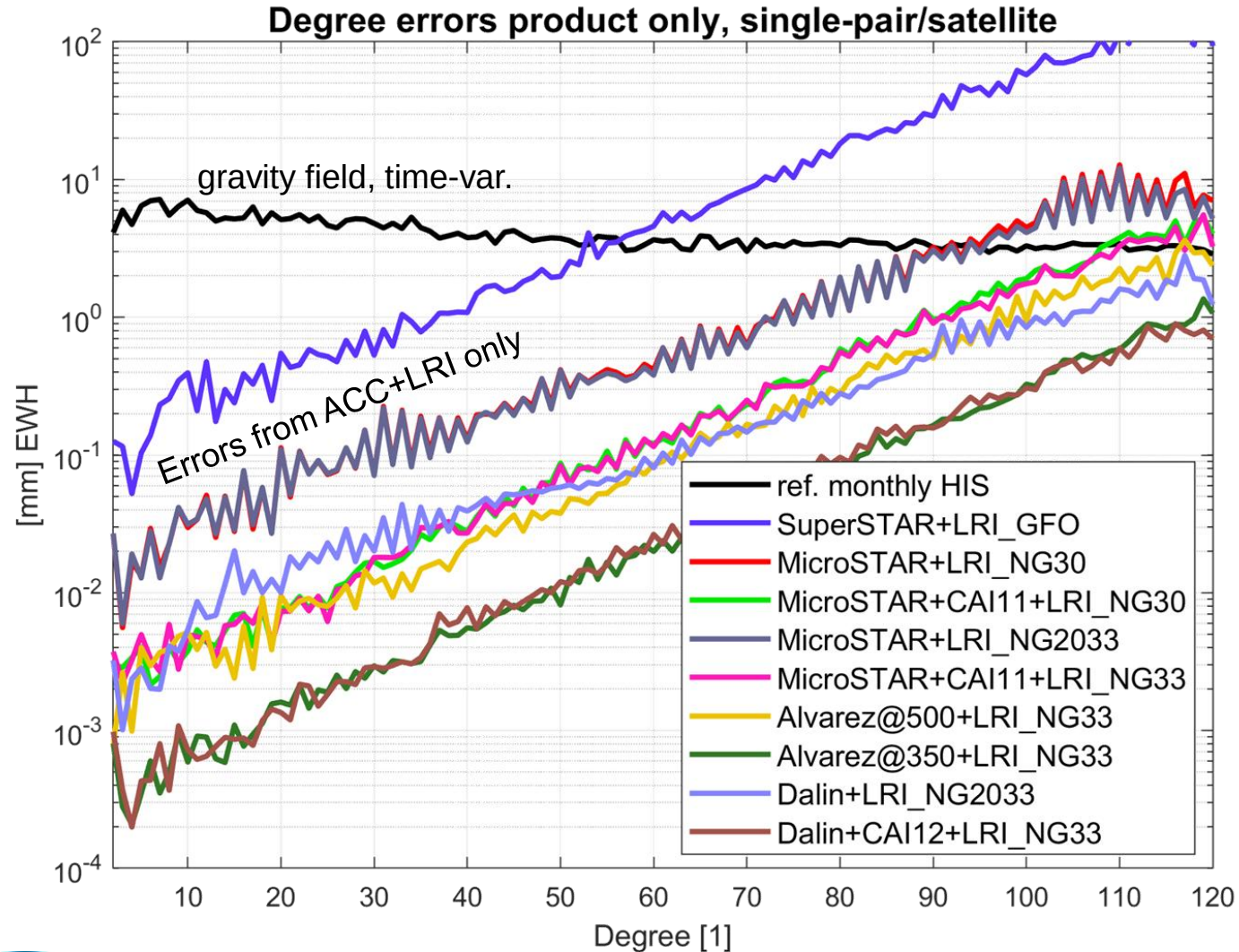
- Single satellite in nadir pointing mode
- High-low satellite-to-satellite tracking (hl-sst) via GNSS



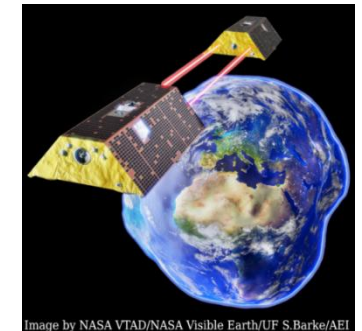
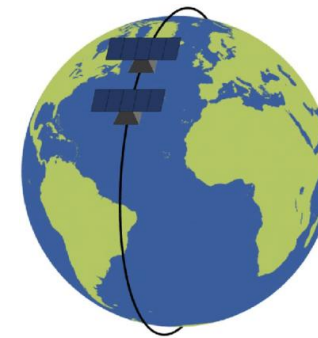
- One CAI accelerometer in along-track direction
- Microgravity allows extended interrogation time
- Performance target specifications
 - Operating performance level: $< 10^{-9} \text{ m s}^{-2} \text{ Hz}^{-1/2}$
 - Quantum projection noise floor: $< 10^{-10} \text{ m s}^{-2} \text{ Hz}^{-1/2}$
- Demonstrate operation of quantum sensors in space, with better performance than on ground

Benefit of CAI ACC in GRACE-like Missions

Polar-pair: (only static gravity as input)

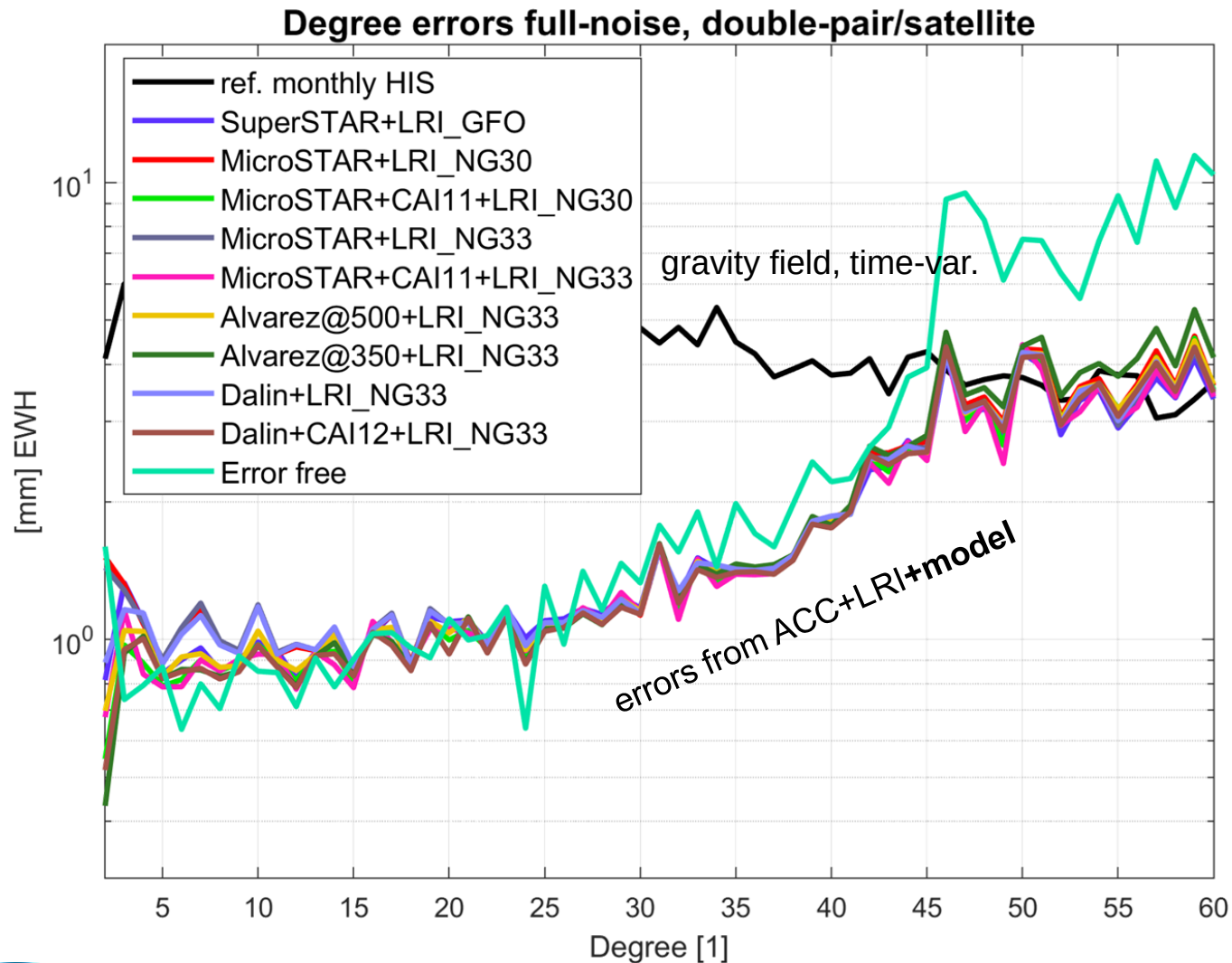


- Retrieval performances scales with instrument performance (ACC, LRI)
- **CAI sensors improve their EA counterparts**

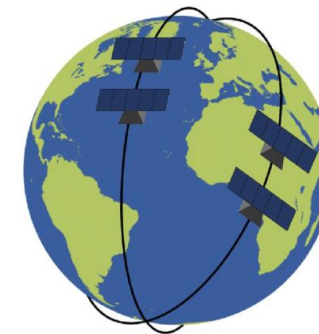


Benefit of CAI ACC in GRACE-like Missions

Results for **full-noise, double-pair**: (with time-variable gravity)

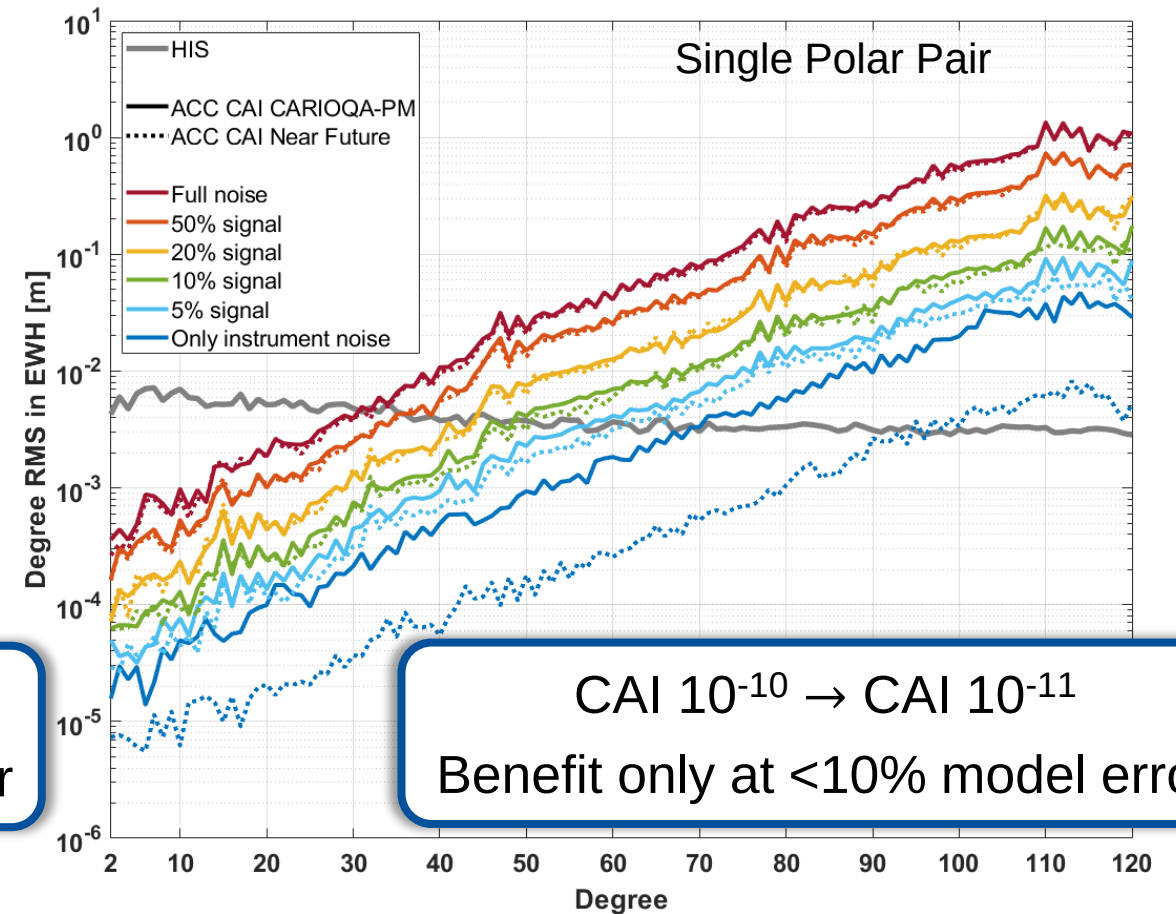
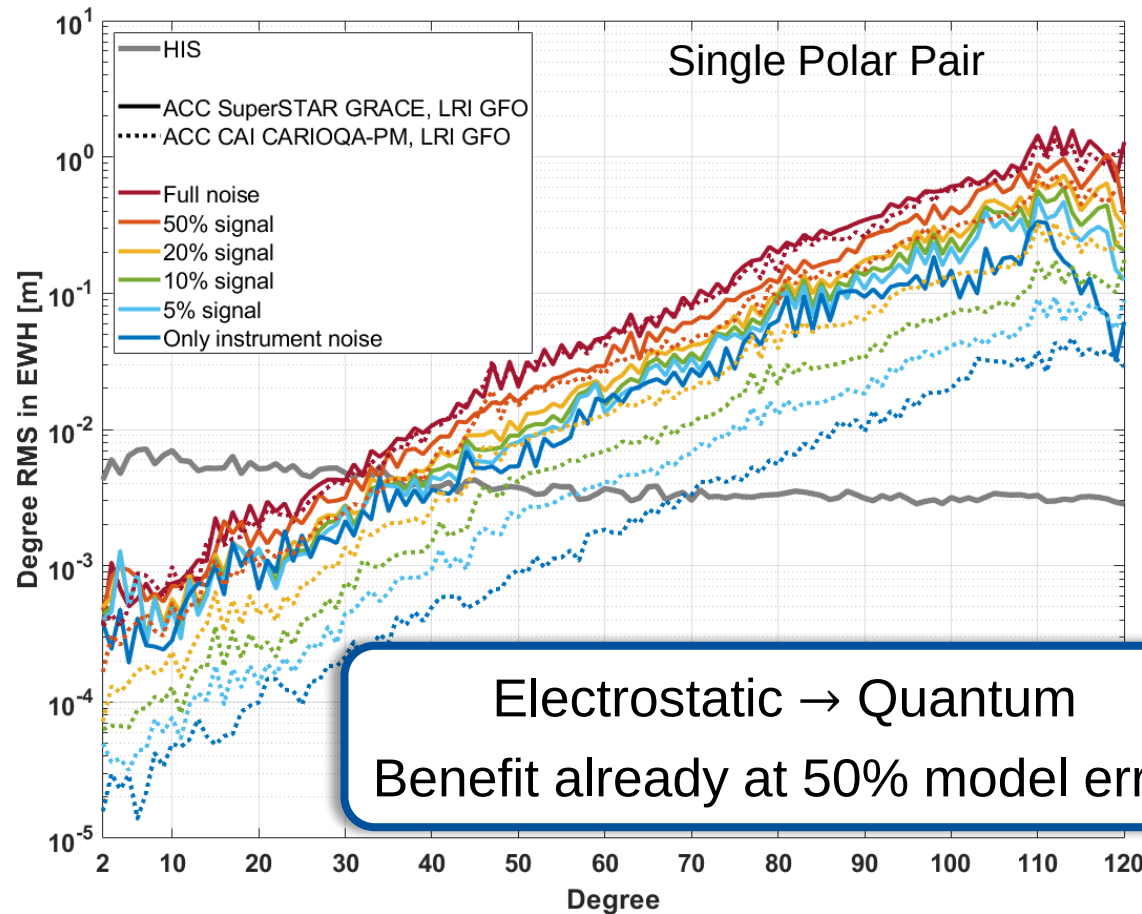


- All models show nearly identical performance
- Temporal aliasing is reduced compared to polar-pair
- But still: **Better handling of temporal aliasing needed, as it is the dominating error source by far**

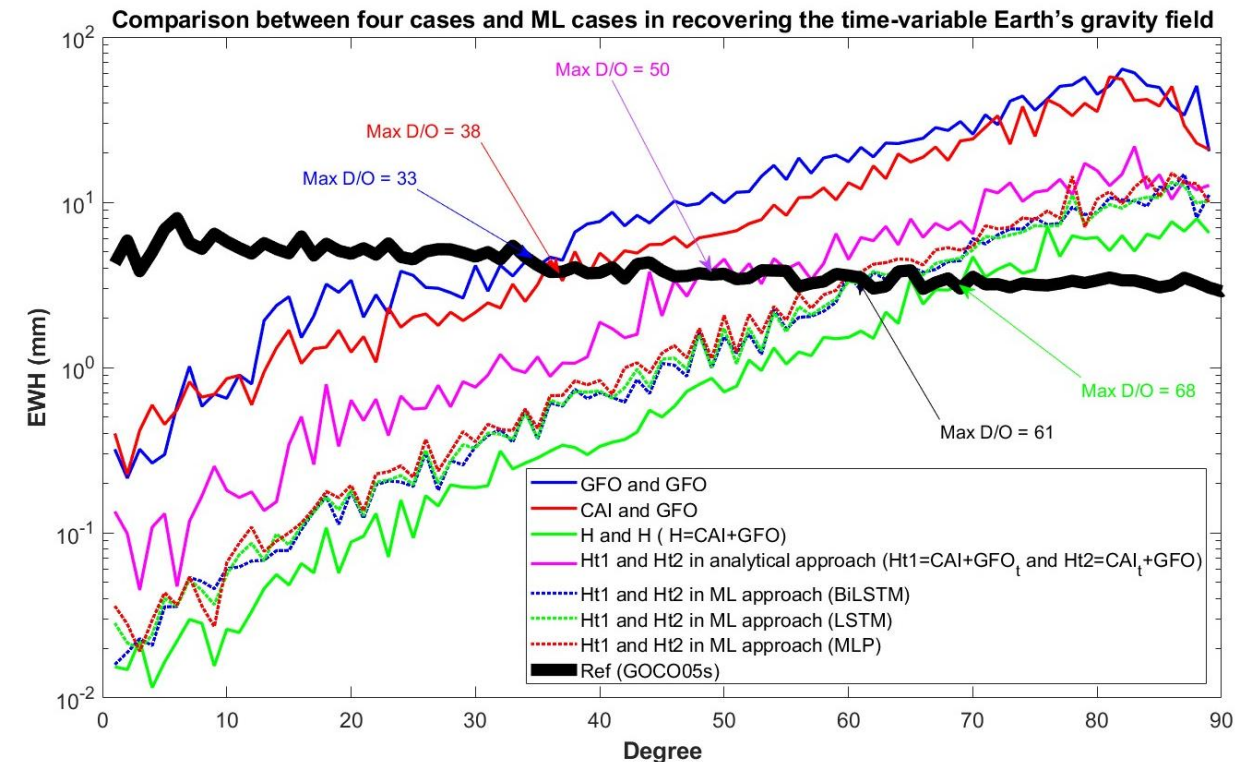
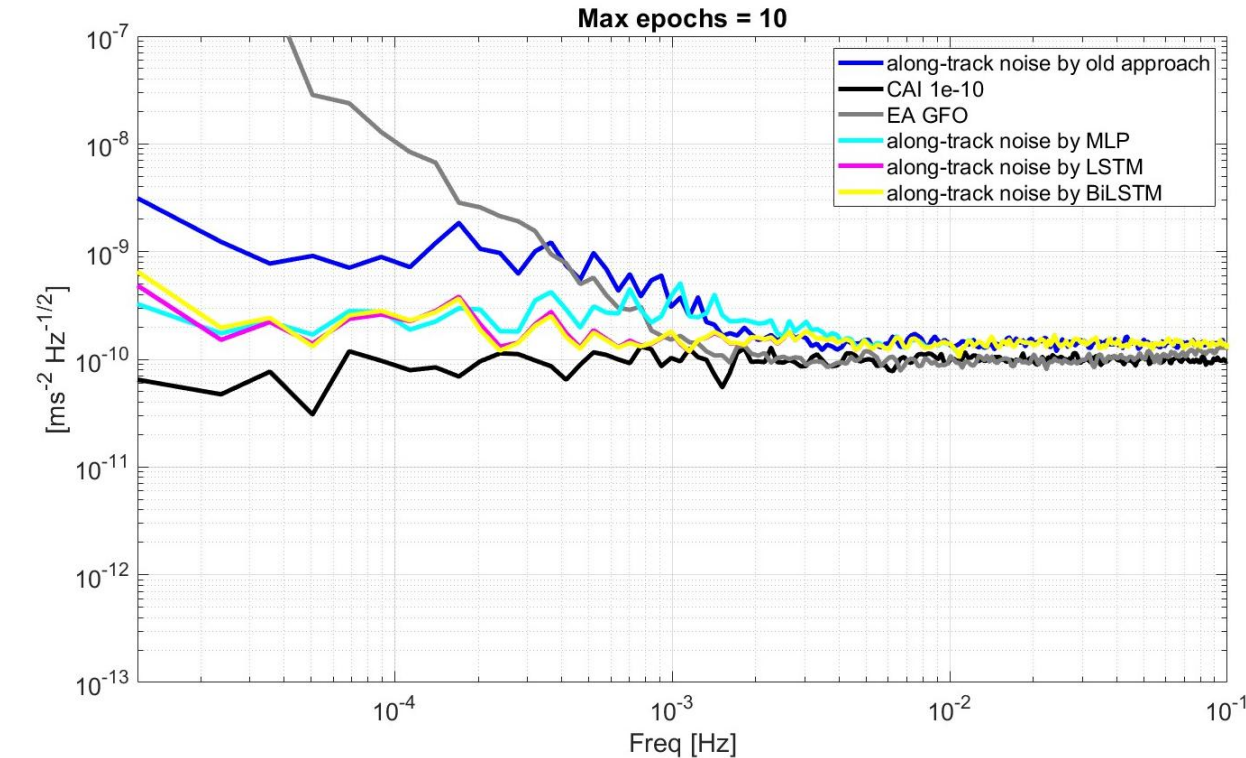
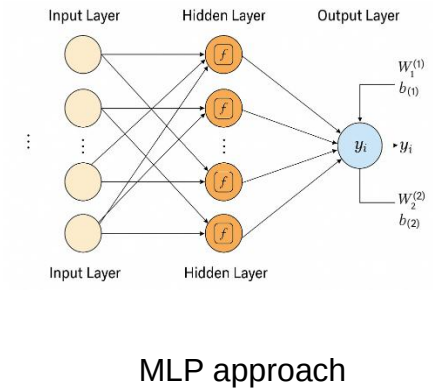
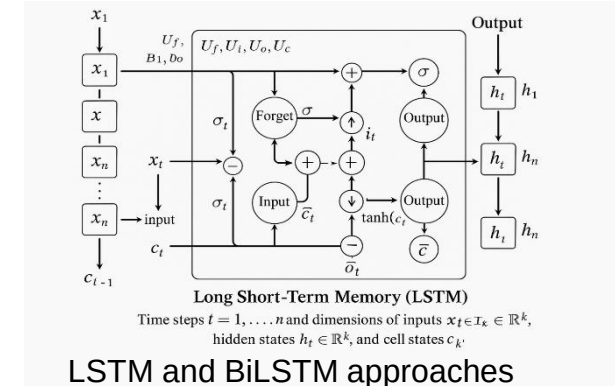
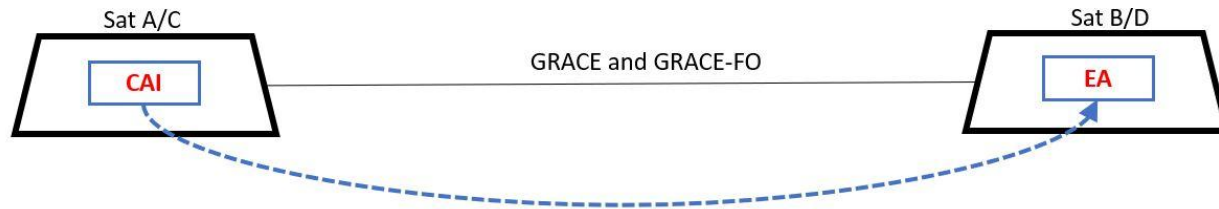


Further Improvement of Background Model Errors is Required

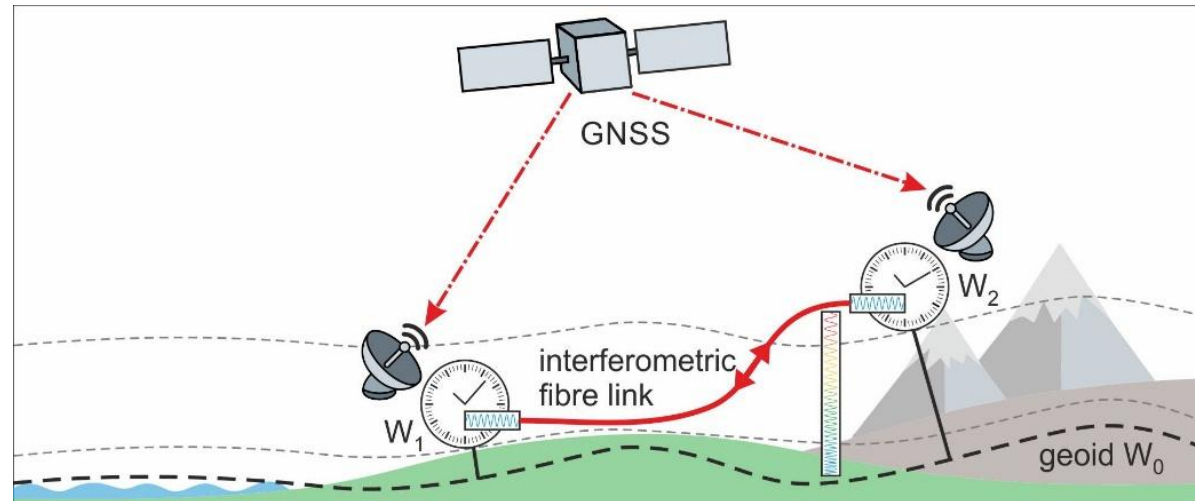
Reduce model errors to determine level where accelerometer accuracy gets significant



Machine Learning for Transplanting Accelerometer Measurements



Relativistic Geodesy with Clocks



Clocks for Determining Physical Heights

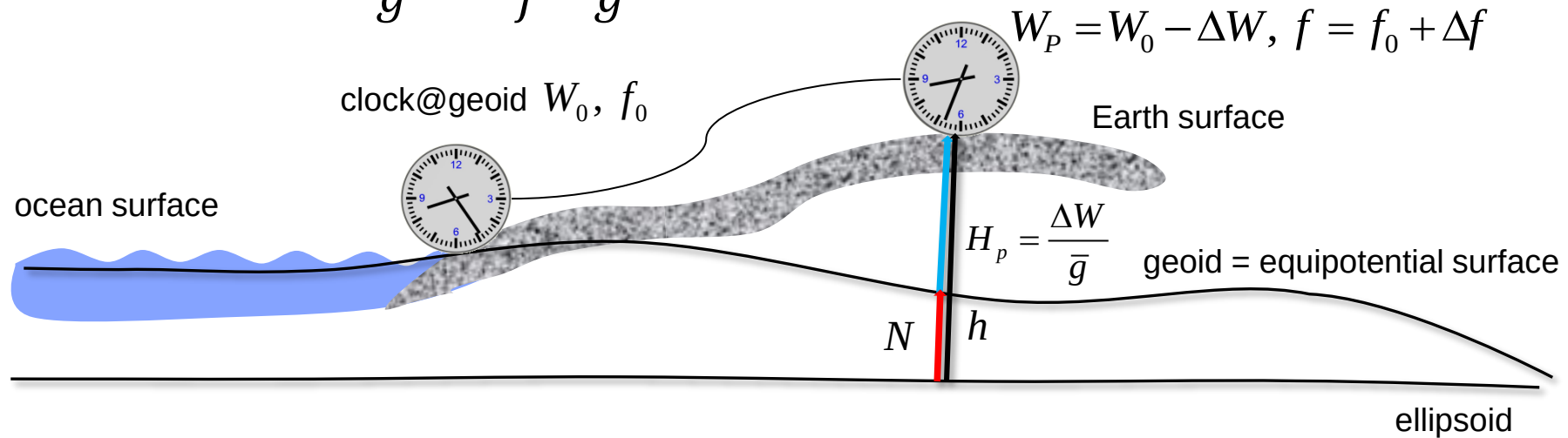
Einstein's theory $\frac{d\tau_1}{d\tau_2} = \frac{df_2}{df_1} = \left(1 - \frac{\Delta W}{c^2}\right)$

clock accuracy: $\frac{\Delta f}{f} (1.0 \times 10^{-18})$
 $\sim \Delta H (1.0 \text{ cm})$



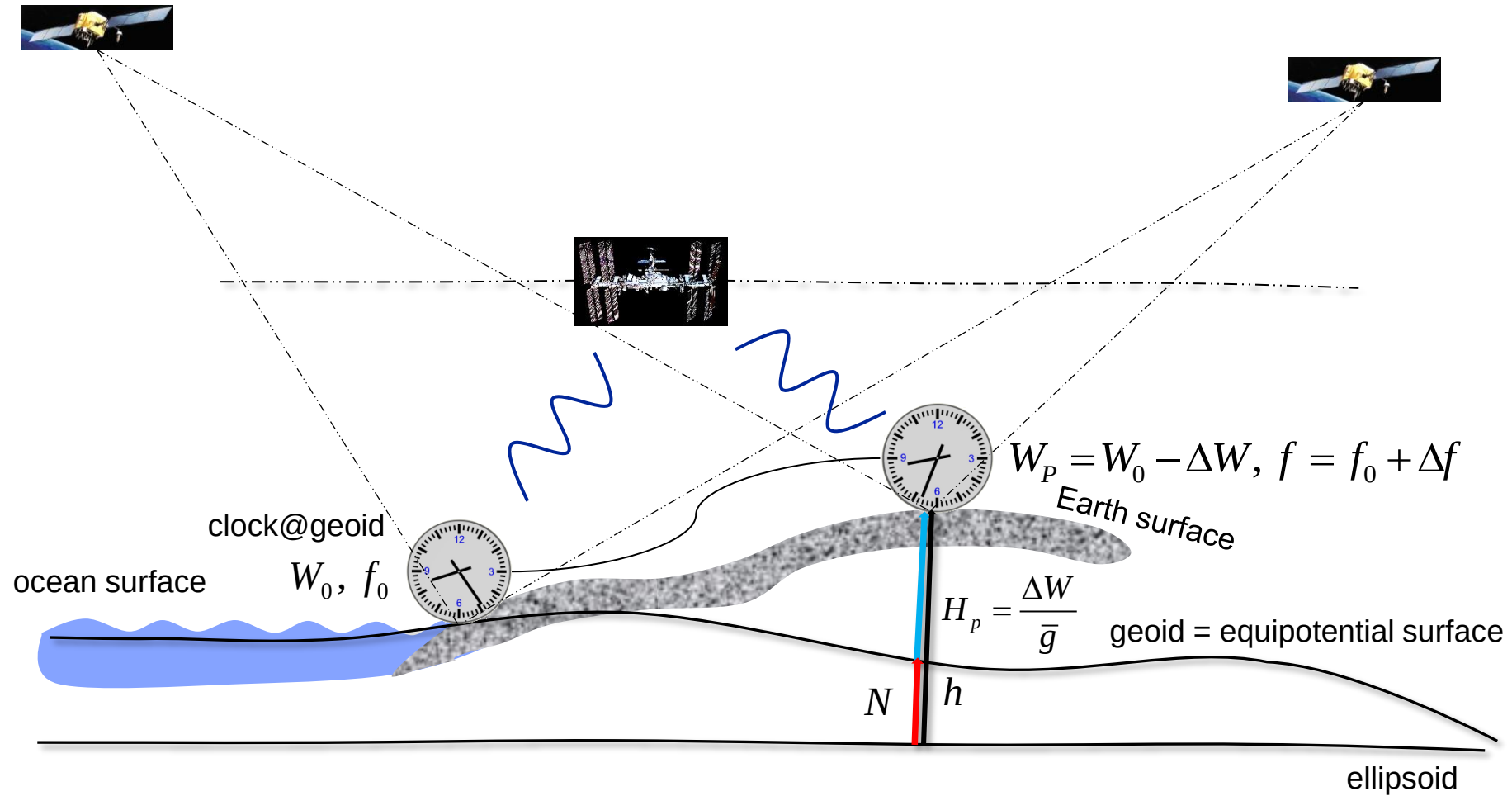
Gravitational redshift $\frac{\Delta f}{f} = \frac{\Delta W}{c^2}$

Physical height $H = \frac{\Delta W}{\bar{g}} = \frac{\Delta f}{f} \frac{c^2}{\bar{g}}$

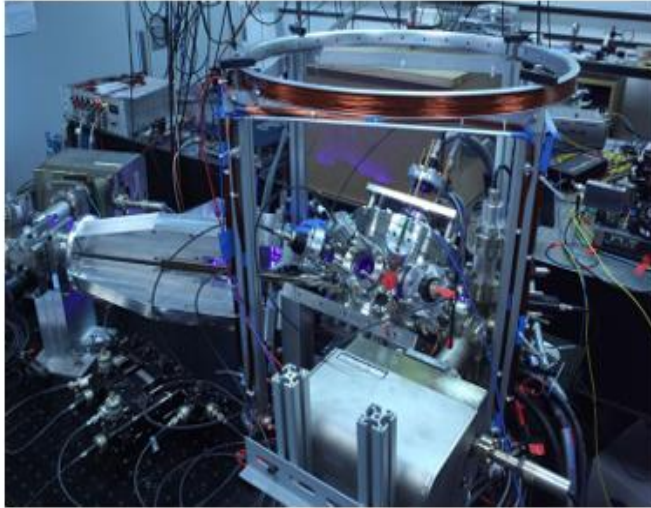


Geoid height $N = h - H$ (h ellipsoidal height from GNSS)

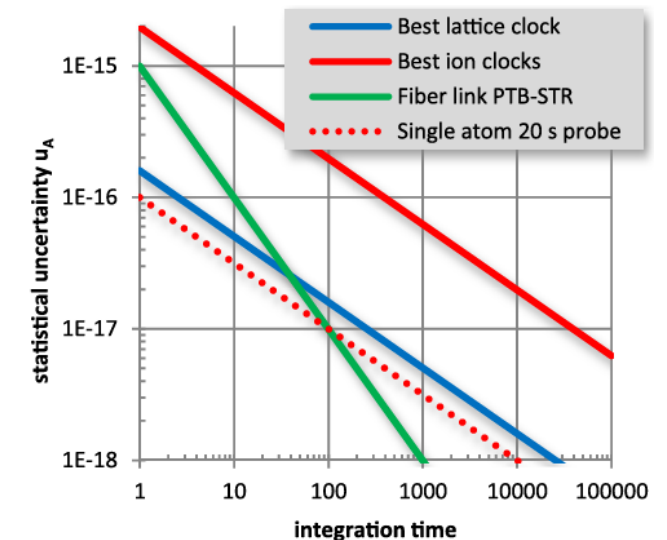
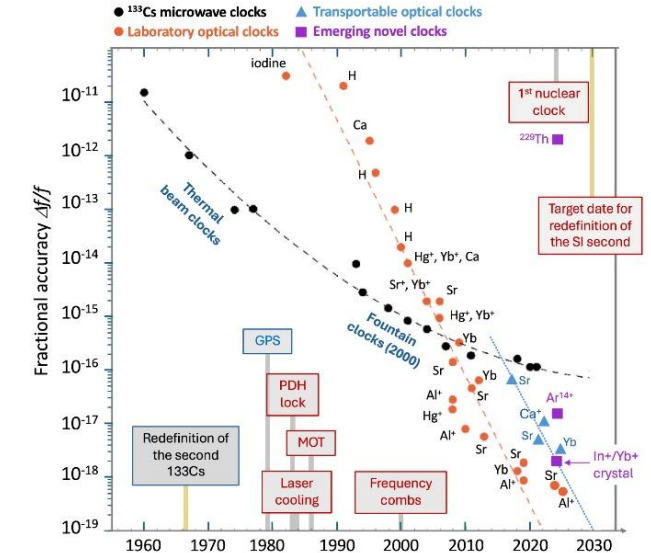
Clocks for Determining Physical Heights - Links



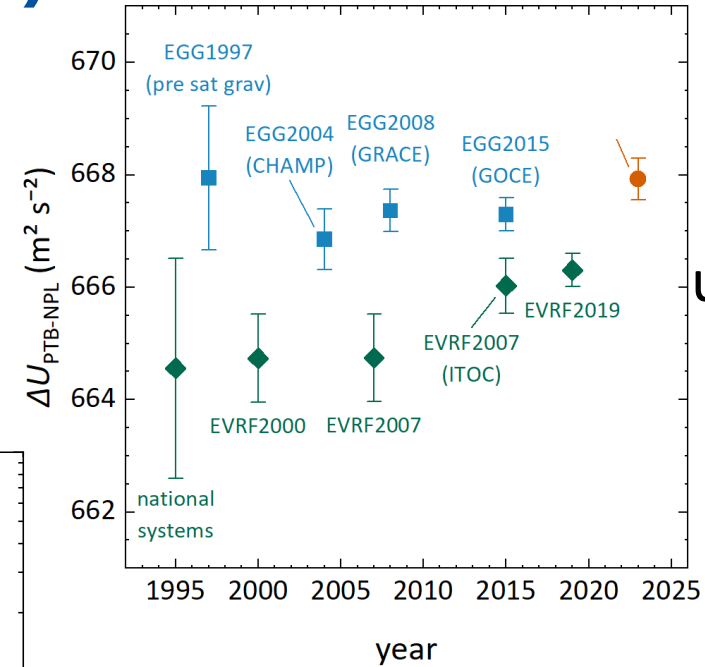
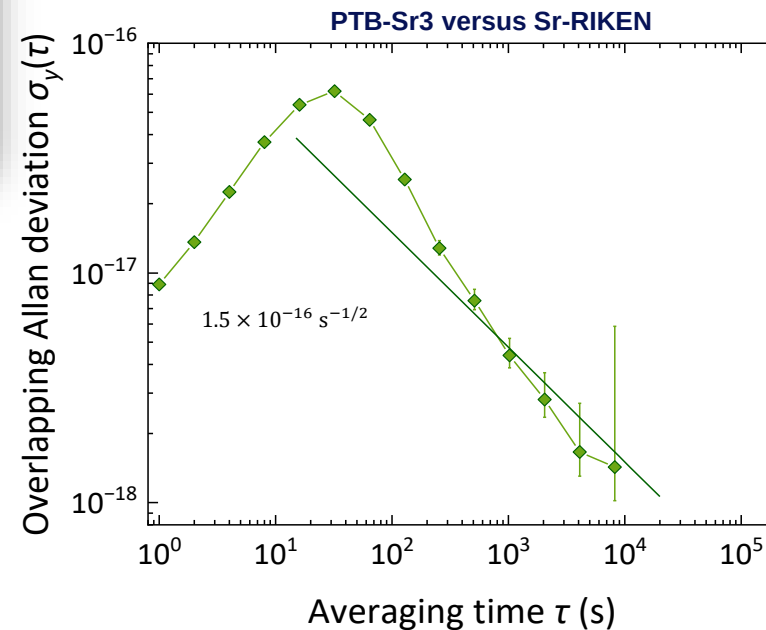
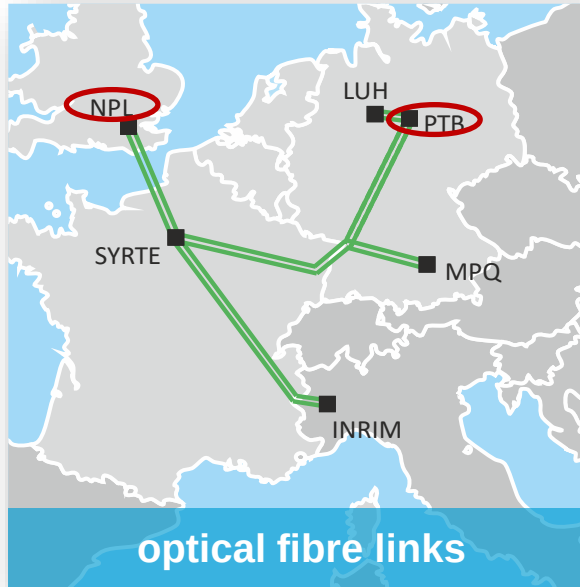
Atomic Clocks & Links



- Statistical **clock** uncertainty decreases with averaging time, enabling higher precision over long observations
- Systematic uncertainties decrease with improved clock design and control of environmental effects
- Optical fibre **links** enable clock comparisons with $\leq 10^{-19}$ fractional uncertainty with sufficient averaging time



Chronometric Levelling Campaign 2023 with Transportable Clocks (RIKEN, PTB)



$u_{\text{CL}} = 3.8 \text{ cm}$

$u_{\text{geod}} = 3.0 \text{ cm}$

Uncertainty of chronometric levelling comparable to established methods

doi.org/10.48550/arXiv.2410.22973



Time Transfer via ACES*

*Atomic Clock Ensemble in Space

Measure time differences over days to weeks to determine height difference ΔH

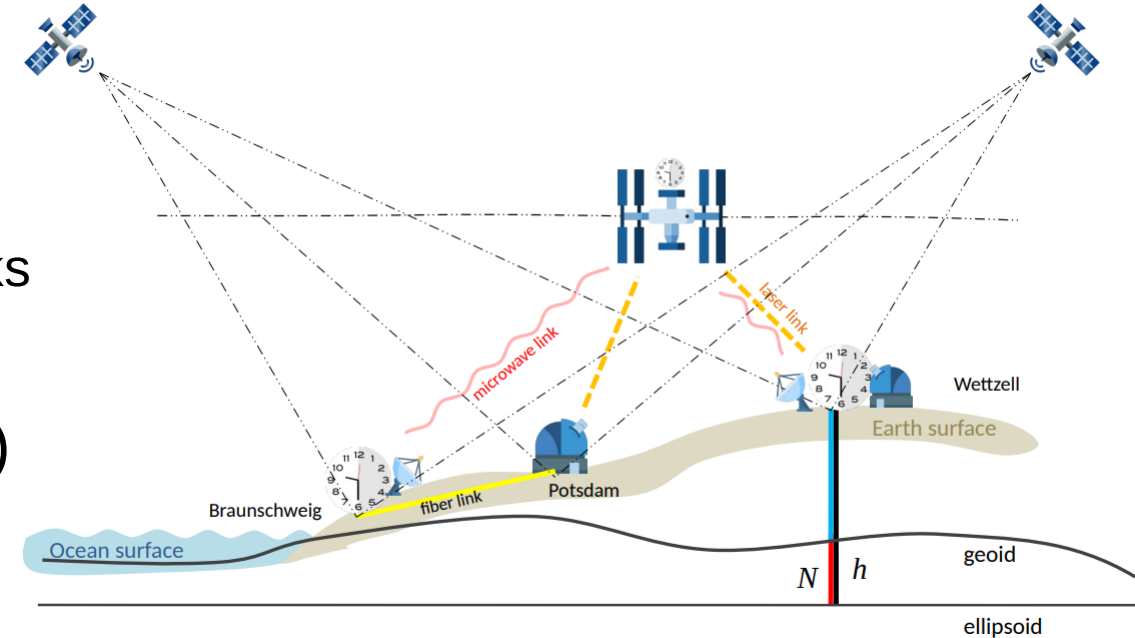
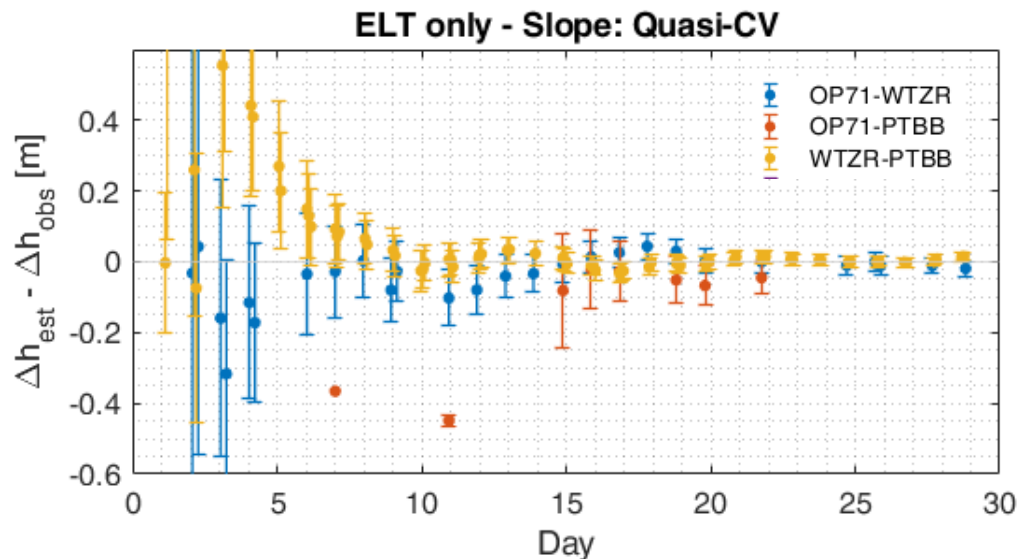
$$\frac{\Delta\tau_{12}}{\tau_2} = \frac{\Delta W_{12}}{c^2} = \frac{\bar{g}\Delta H_{12}}{c^2}$$

where ΔW_{12} gravity potential difference

$\tau_2 - \tau_1$ measured time difference of the clocks

$\bar{g}$ mean gravity at each clock site

Assumptions on clock and link noise ($\sim 10^{-17}$)



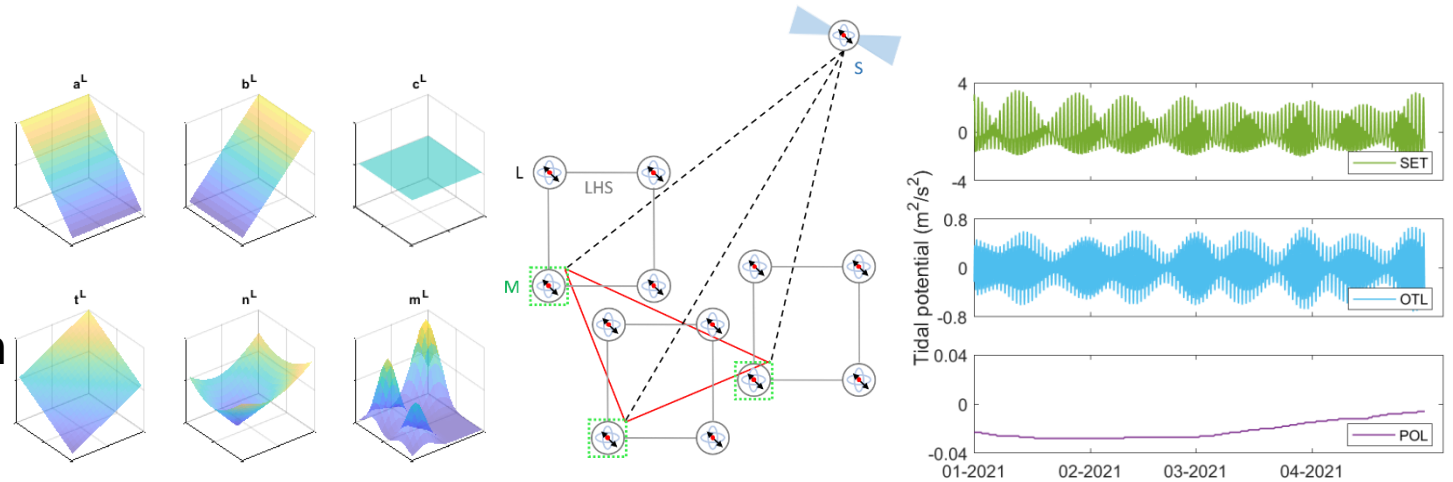
Lachmann (2025)

20

Height System Unification - Globally

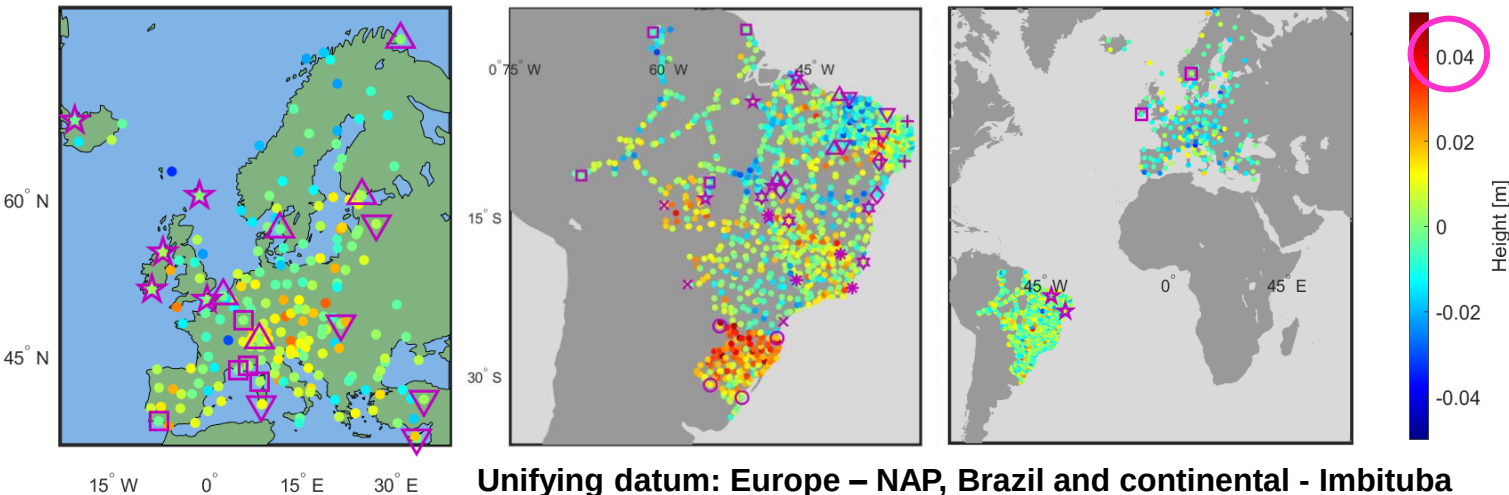
Estimation of various error parameters in local height systems like offsets, tilts, specific systematics, etc.

Achievable **unification accuracy** < 2 cm with proper distribution of clocks



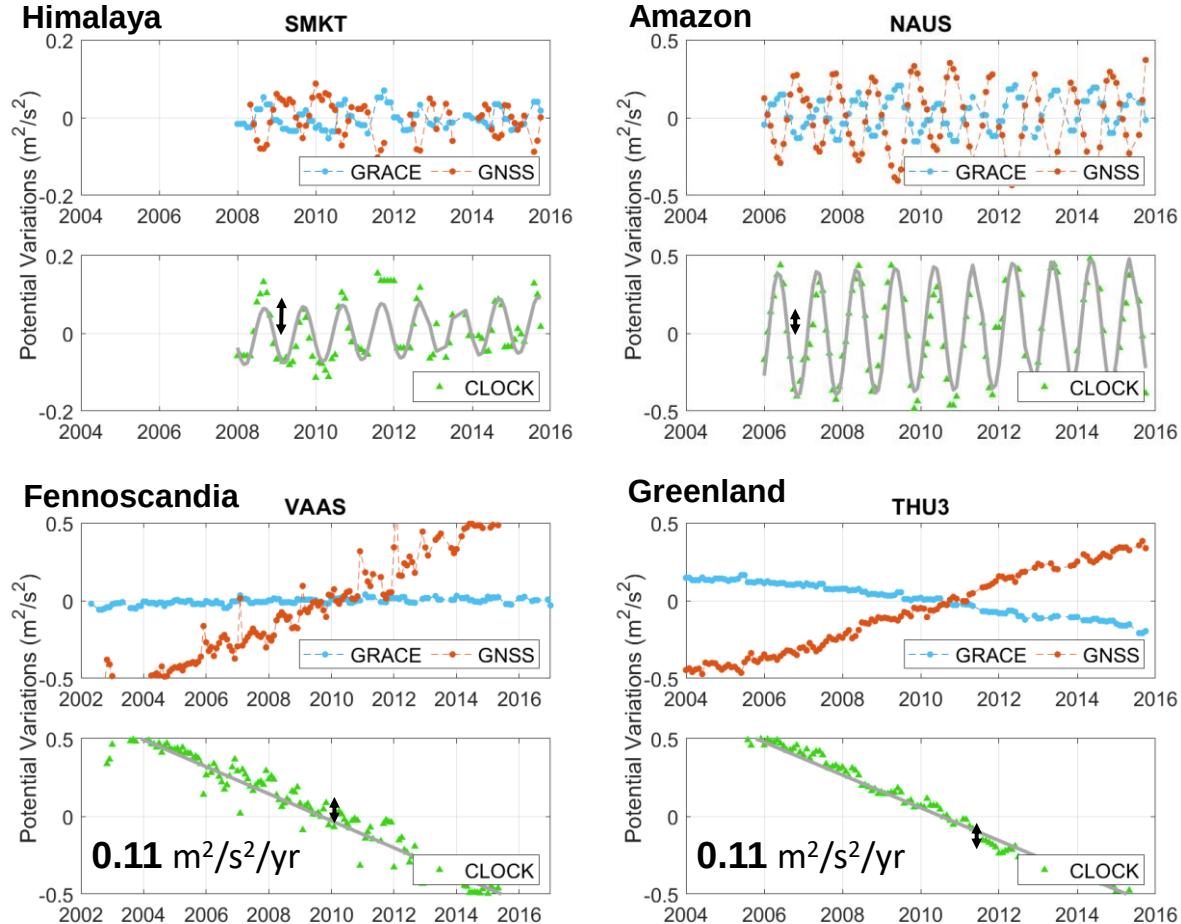
Symbolic representation of error parameters in a local height system, link configuration, time series of tidal effects: SET (solid-earth), OTL (ocean load) and pole tide (POL)

Adjusted error after reunification processes



- Reduced links with master clocks and local clocks
 - Errors/uncertainties
 - Tidal model: 10^{-18}
 - Clocks: $10^{-18} - 10^{-17}$
 - Space link: 10^{-18}
 - Fiber link: 10^{-19}
- Unification to global geoid with a known offset

Time-Variable Gravity Signals Detection



Seasonal signals

- Himalaya: Maximum range of $[-0.2 \ 0.2] \text{ m}^2/\text{s}^2$
- Amazon: Maximum range of $[-0.5 \ 0.5] \text{ m}^2/\text{s}^2$

Long-term signals

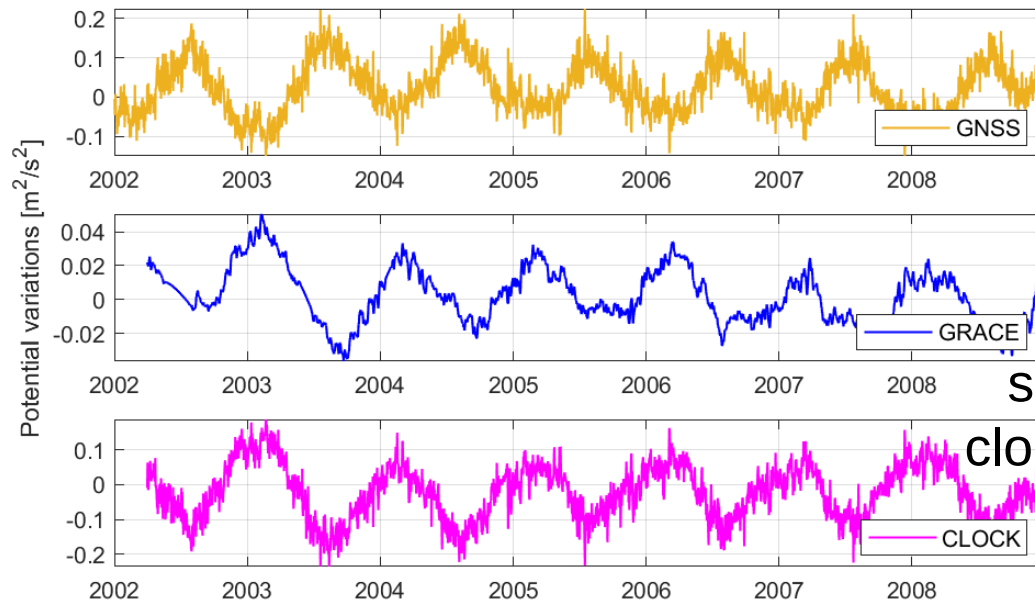
- Greenland (present-day ice-mass loss): $1 \text{ m}^2/\text{s}^2$ between 2004-2015
- Fennoscandia (GIA): 1 cm/yr uplift near land uplift center

Clock networks are complementary to satellite results (like GRACE-FO):

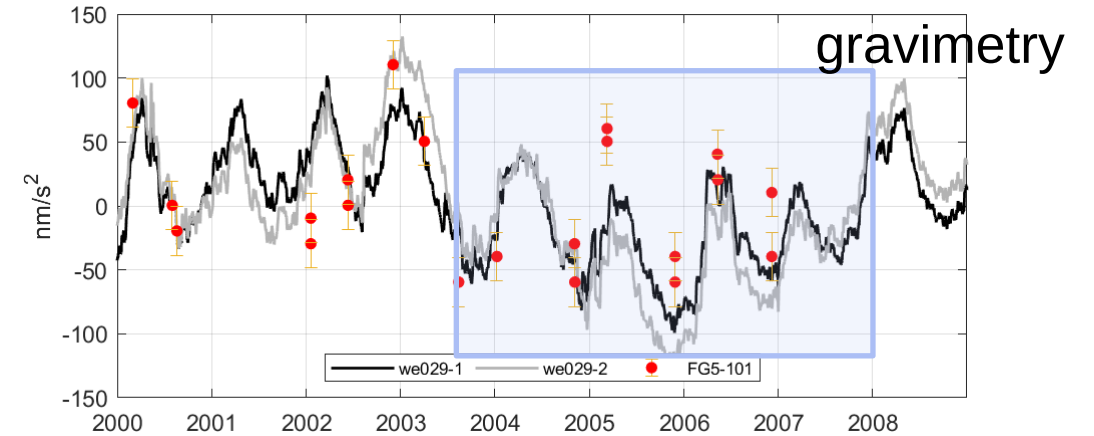
- Point-wise and high-frequency sampling observations
- Near real-time

$$\text{clock accuracy: } \frac{\Delta f}{f} = 1 \times 10^{-18} \sim \Delta H = 1 \text{ cm} \sim \Delta V = 0.1 \text{ m}^2/\text{s}^2$$

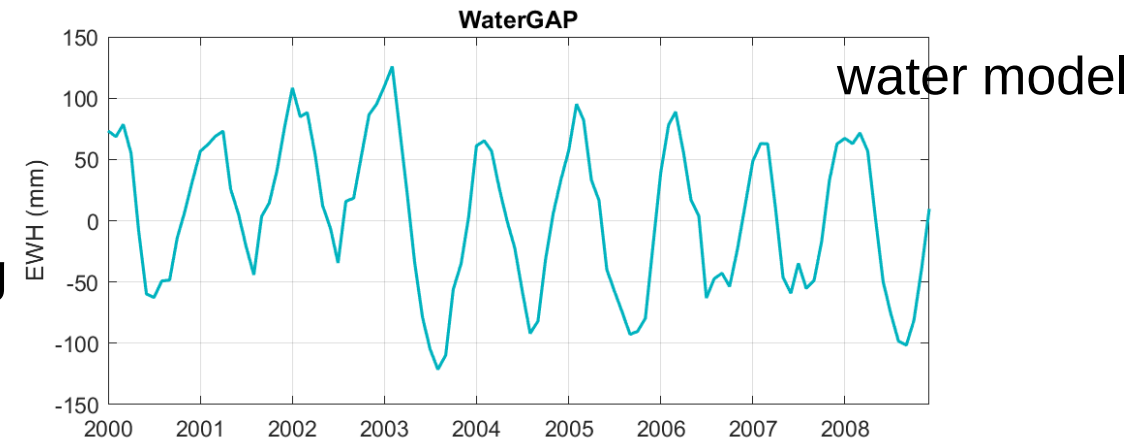
Temporal Variations at Wettzell



GNSS, GRACE and simulated clock potential variations (daily averaged)



Terrestrial gravity observations SG dual sphere and AG (daily averaged)



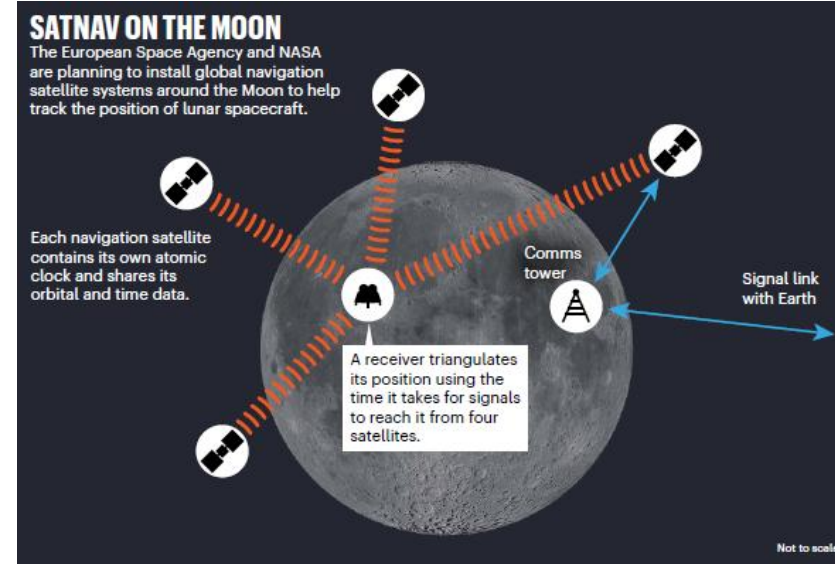
WaterGAP TWS in terms of EWH ($0.5^\circ \times 0.5^\circ$; monthly)

- Potential decrease after the dry summer of 2003, followed by recovery of local water storage over the next three years, is observed in GRACE, GNSS, and simulated clock data
- SG gravity residuals show interannual variations from 2003 to 2007, indicating additional local-scale variability

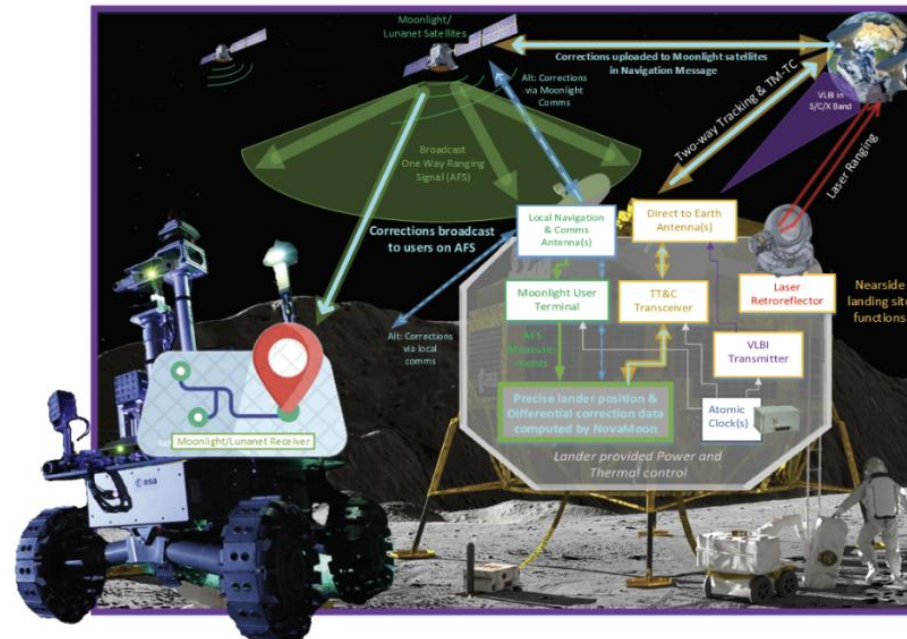
Time System on the Moon

- ❑ lunar clock as possible reference clock
 - applications in chronometric geodesy
 - geopotential determination
 - height system unification
 - sea level monitoring, etc.
 - lunar velocity well known by LLR, less environmental noise on the Moon

- ❑ lunar independent time system
 - future lunar mission plans, projects like ESA's Moonlight and NASA's LCRNS
 - link to terrestrial time standards
 - IAG WG (led by A. Fienga)



Gibney (2023)



NovaMoon concept from Molli et al. (2026)

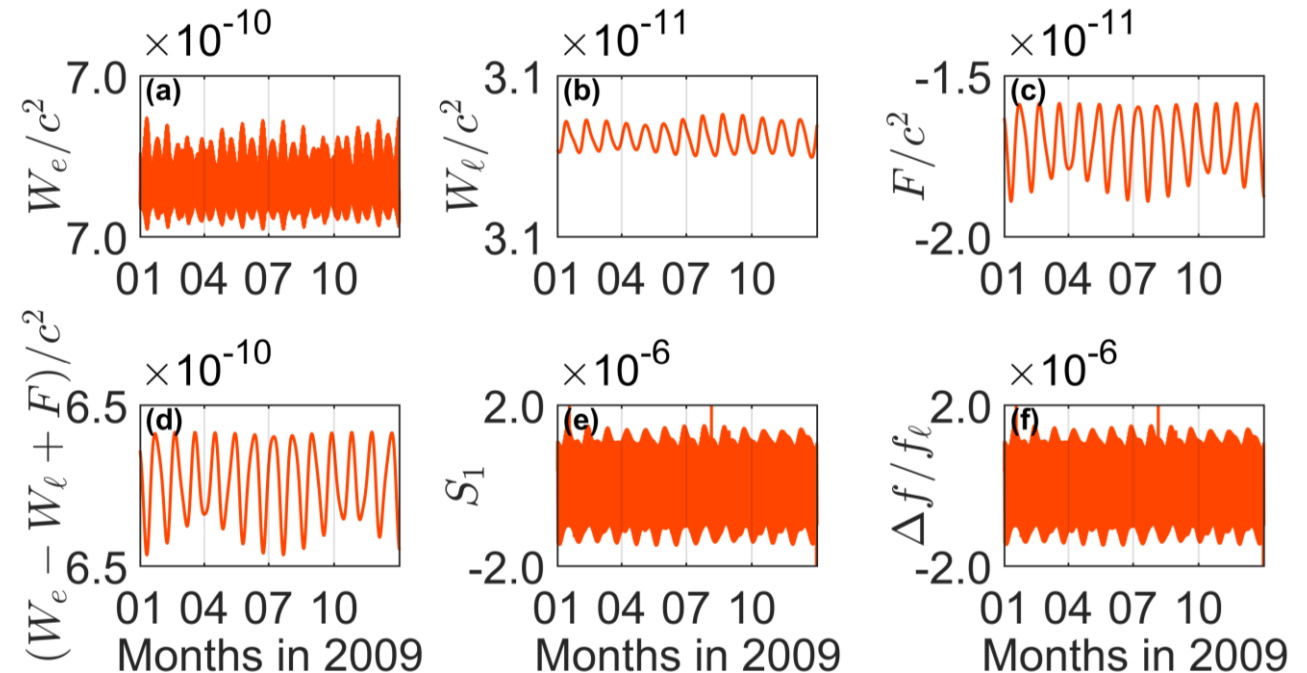
Earth-Moon Clock Frequency Difference

- fractional frequency difference model of clocks on Earth (E) and the Moon (L)

$$\frac{f_E - f_L}{f_L} = \frac{W_E - W_L}{c^2} + \frac{F}{c^2} + S_1 + C_1 + O(c^{-4})$$

- $\frac{W_E - W_L}{c^2}$: contribution from gravity potential difference of E- and L-clocks
- $\frac{F}{c^2} = \frac{dTCL}{dTCG} - 1$: coordinate time relation between Earth and Moon local systems (TCL , TCL - Geocentric and Lunar Coordinate Times)
- S_1 : contribution from signal propagation (Doppler, atmosphere, Shapiro)
- $C_1 = S_1 \frac{W_E - W_L + F}{c^2}$

- LLR provides accurate lunar position, velocity, acceleration, and angular velocity, etc.
- assumed Earth clocks at LLR observatories
- assumed Lunar clocks at reflector positions
- assumed observation uncertainty: 10^{-18} (~1 cm height change on Earth and ~6 cm on the Moon)



Conclusions

Quantum metrology provides novel methods for gravimetric Earth observation

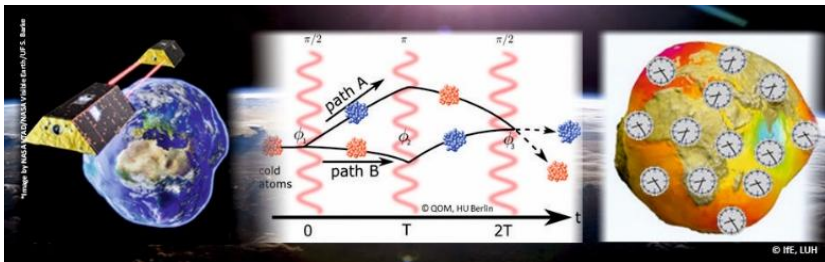
Beneficial use of novel sensors

- **Quantum gravimeters** on ground enable new applications in geosciences
- (Hybrid) **atom interferometric** accelerometers **in space** missions, but one could only partly exploit them at the moment

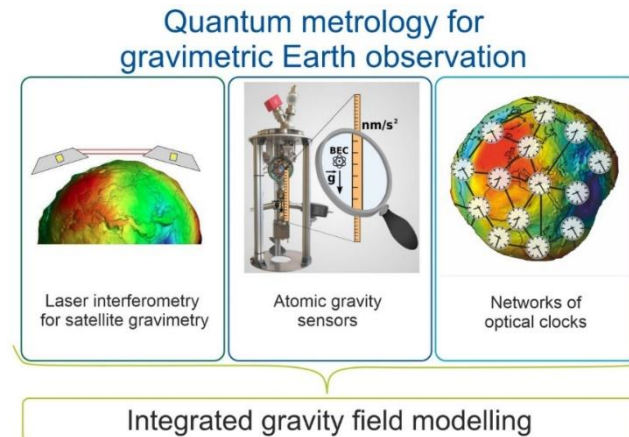
Insufficient background modelling has major impact on gravity field solution

→ Way out: more satellites, intelligent parameterization, improved models, ...

- Relativistic geodesy with **clocks for height systems** and global reference (+ Moon)
- Clocks **networks** for monitoring of **mass variations**



IAG: Quantum Sensors for Geodesy
<https://quge.iag-aig.org>



SFB 1464 TerraQ: Relativistic and Quantum-based Geodesy
www.terraq.uni-hannover.de

